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Underground structure detection by surface magnetic gradient measurements

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ABSTRACT

This problem involves magnetic induction methods to locate and determine the depth of a subsurface line source of magnetic field. The origin of the field may be self-generated or induced by a surface transmitter. The experimental method requires measuring the horizontal gradient of either the vertical or horizontal component of the field rather than the field itself so as to increase signal to noise ratio. A mathematical outline is presented and experimental results are discussed.

Keywords: underground detection, electromagnetic theory, magnetic induction, tunnels, underground structures, imaging

1. INTRODUCTION

It has been experimentally demonstrated by Sandia, LANL, and Stolar Research Corp. that underground structures may be detected by a surface measurement of the horizontal gradient of either the vertical or horizontal component of the magnetic field originating at the structure location. What follows is an outline of the theory serving as a basis of the location method. Not only is location information contained in the data, but so also is structure depth. The particular problem attacked here is that of a long conductor such as a pipe, wire, or railroad track in a tunnel.[1,2]

The reason for measuring the gradient (actually a component of the gradient) is that, whereas the wavefront signal from the buried line has a non-zero curvature, hence a non-zero field gradient, the wavefronts from potential distant surface sources are plane with zero gradient. Thus, a weak change in the target generated magnetic field in the presence of possibly a much stronger field is detectable where the target field itself may be lost in the noise.

The origin of the target magnetic field may be passive or active. The passive field originates in currents generated in the structure (tunnel) itself, perhaps by internal generators and distributed by wiring and transmission lines. The active field (as experimentally used by Stolar) originates by a surface transmitter which propagates waves into the earth that scatter off the structure.[3] That part which is backscattered to the surface then constitutes the return active field.

The mathematical outline begins by assuming a line current I buried in the earth with index of refraction n at a depth y below the surface. See Figure 1. The angles of incidence and refraction are θ and α respectively. The index, n, is in general a complex number so as to cover the realistic case of an attenuating earth. χ measures the horizontal distance from the surface trace of the current source. At the low frequencies used in the experiments, conduction current in the earth dominates displacement current.

The outline below is a result of Maxwell's equations. For example, Snell's law comes directly from an application of Maxwell's equations by applying the boundary conditions to a surface separating media with different indices of refraction. Also recall that the index of refraction follows from a knowledge of the dielectric constant, magnetic permeability and conductivity.

2. BOUNDARY CONDITIONS

The phases of the incident, reflected and refracted waves must match at all points on the plane of separation (earth's surface) at all times t. This requires the following conclusions:

1. The frequency remains invariant.

2. Law of reflection (k vectors all lie in the same plane with equal reflection and incident angles).

3. Snell's law: $n \sin\theta = (1)\sin\alpha$

In addition, the tangential components of E and H, and the normal components of D and B must match. Only two of the four are independent. Thus, only continuity of E_{tan} a H_{tan} are required. Once those equations are solved, the use of Snell's law and some trig identities produce the equations below.

3. OUTLINE OF DERIVATION

$$E = E_i \frac{2\cos\theta \sin\alpha}{\sin(\theta+\alpha)} \tag{1}$$

a Fresnel equation,[4] where E_i=incident field just below the surface.

Then $B = B_i \dfrac{2\cos\theta\sin\alpha}{n\sin(\theta+\alpha)}$, (2)

and from $\oint B\cdot dl = \mu_0 I$, $B_i = \dfrac{\mu_0 I}{2\pi\left(x^2+y^2\right)^{\frac{1}{2}}}$. (3)

We really want $\dfrac{\partial B_x}{\partial x}$, holding y constant. One way to do this is to eliminate θ and α in favor of x and y via

$$\sin\alpha = n\sin\theta, \tag{4}$$

$$\cos\alpha = \left[1 - n^2\sin^2\theta\right]^{\frac{1}{2}}, \tag{5}$$

$$\sin\theta = \frac{x}{\left(x^2+y^2\right)^{\frac{1}{2}}}, \tag{6}$$

$$\cos\theta = \frac{y}{\left(x^2+y^2\right)^{\frac{1}{2}}} \tag{7}$$

The result is $B_x = \left(\dfrac{\mu_o I}{\pi n}\right)\dfrac{1}{\left(x^2+y^2\right)\left[\dfrac{1}{ny} + \dfrac{1}{\left[\left(1-n^2\right)x^2+y^2\right]^{\frac{1}{2}}}\right]}$, then (8)

$$\frac{\partial B_x}{\partial x}=-\left(\frac{\mu_0 I}{\pi n}\right)\left(\frac{x}{x^2+y^2}\right)\frac{\left(\frac{2}{x^2+y^2}\right)\left(\frac{1}{ny}+\frac{1}{\left[\left(1-n^2\right)x^2+y^2\right]^{\frac{1}{2}}}\right)-\frac{\left(1-n^2\right)}{\left[\left(1-n^2\right)x^2+y^2\right]^{3/2}}}{\left[\frac{1}{ny}+\frac{1}{\left[\left(1-n^2\right)x^2+y^2\right]^{\frac{1}{2}}}\right]^2} \tag{9}$$

Note: for n=1(free space), $\frac{\partial B_x}{\partial x}=-\frac{\mu_0 I}{\pi}\left[\frac{xy}{\left(x^2+y^2\right)^2}\right]$

(10)

This free space equation from (10) is plotted in Figure 2 for y=40m; the vertical scale is arbitrary.

The following points are noted:

1. The x coordinate of the peaks is always $y/\sqrt{3}$, so that for this simple example, the depth of the current source is given by $\sqrt{3}x_{m,}$ where x_m is the horizontal distance to the max of $\left|\frac{\partial B_x}{\partial x}\right|$.

2. $\left|\frac{\partial B_x}{\partial x}\right|$ should be zero directly over the source.

In a similar fashion to deriving B_x and $\frac{\partial B_x}{\partial x}$, B_y and $\frac{\partial B_y}{\partial x}$ may be found.

$$B_y=-B\sin\alpha=B_i\frac{2}{n}\frac{\cos\theta\sin^2\alpha}{\sin(\theta+\alpha)},\ \text{thus} \tag{11}$$

$$B_y=-\frac{\mu_o I_n}{\pi}\left(\frac{1}{x^2+y^2}\right)\frac{xy}{\left[\left(1-n^2\right)x^2+y^2\right]^{\frac{1}{2}}+ny} \tag{12}$$

$$\frac{\partial B_y}{\partial x}=-\left(\frac{\mu_0 nI}{\pi}\right)\left(\frac{y}{x^2+y^2}\right)\frac{1}{\left[\left(1-n^2\right)x^2+y^2\right]^{\frac{1}{2}}+ny}\left[1-\frac{2x^2}{x^2+y^2}\frac{\left(1-n^2\right)x^2}{\left\{\left[\left(1-n^2\right)x^2+y^2\right]^{\frac{1}{2}}+ny\right\}\left[\left(1-n^2\right)x^2+y^2\right]^{\frac{1}{2}}}\right] \tag{13}$$

$$\text{Note that for } n=1, \left|\frac{\partial B_y}{\partial x}\right|=\frac{\mu_0 I}{2\pi}\left|\frac{x^2-y^2}{\left(x^2+y^2\right)^2}\right|. \tag{14}$$

This free space equation from (14) is plotted in Figure 3 for y=40m with an arbitrary vertical scale.

Note that $\left|\frac{\partial B_y}{\partial x}\right|$ is a max directly over the current and that it becomes zero at a horizontal distance equal to the depth. That is, the depth = the horizontal distance to where $\left|\frac{\partial B_y}{\partial x}\right|$ goes to zero.

4. EFFECT OF TOTAL INTERNAL REFLECTION[5]

Recall that when the incident angle θ (in the earth) reaches the critical angle, θ_c, the refracted wave is horizontal with $\alpha = 90^0$. This occurs for $\sin\theta_c = \frac{1}{n}$. For $\theta > \theta_c$ the *steady state* average has all of the energy reflected at the surface back into the earth with equal reflection and incident angles. Note that there's no discontinuity at $\theta = \theta c$. As θ becomes larger (for $\theta < \theta_c$), the reflected beam grows stronger while the refracted beam grows weaker.

For all incident angles which exceed the critical angle, the excitation in the second medium (air) consists of an inhomogenous plane surface wave propagating in the x direction with an amplitude which falls off exponentially in the y direction (measured now from the earth's surface up), namely,

$$e^{-k\left[\left(\frac{\sin\theta}{\sin\theta_c}\right)^2-1\right]^{\frac{1}{2}}y}\; e^{i\left[k\frac{\sin\theta}{\sin\theta_c}x-\omega t\right]}$$

Notice that for small earth conductivity, surfaces of constant phase (wave fronts) are perpendicular to surfaces of constant amplitude. There is no corresponding x propagating wave in the earth. The amplitude drops off 1/e of its surface value at a vertical distance

$$\frac{\lambda}{2\pi\left[n^2\frac{x^2}{y^2}-1\right]^{\frac{1}{2}}},$$

where λ is the wavelength in air. The phase velocity of the inhomogenous surface wave is

$$v = \frac{c}{n}\left[1+\left(\frac{y}{x}\right)^2\right]^{\frac{1}{2}}, \tag{15}$$

so that waves which emerge at increasing x move slower.

An analysis shows that, for distances $x > \frac{y}{\sqrt{n^2-1}}$, $\left|\frac{\partial B_y}{\partial x}\right|$ may be more useful than $\left|\frac{\partial B_x}{\partial x}\right|$, from an experimental viewpoint. Furthermore, since the line source is a finite distance below the earth's surface rather than infinite, such as for an incident plane wave, some portion of the cylindrical wavefront will always correspond to $\theta \langle \theta_c$, whereas, all portions of a plane wave have the same value of θ.

5. PHASE

The phase difference just at the surface between the wave at x = 0 (directly over the target) and at any arbitrary distance x perpendicular to the line is

$$\delta = \frac{2\pi ny}{\lambda}\left[\sqrt{1+\left(\frac{x}{y}\right)^2} - 1\right] \text{ radians,} \tag{16}$$

where y is the target depth and λ is the free space wavelength. Note that far from the target compared to the depth (x>>y), δ reduces to

$$\delta = \frac{2\pi n}{\lambda} x, (x \rangle\rangle y), \tag{17}$$

which is linear in x and shows that the difference continually increases with x without any maximum or minimum. Close in (x<<y), δ reduces to

$$\delta = \frac{\pi n x^2}{\lambda y}, (x \langle\langle y), \tag{18}$$

Can the depth, y, be determined from a measurement of $\delta(x)$ at an arbitrary position x? The answer is yes, namely

$$y = \frac{xn}{\delta\lambda} x^2 - \frac{\delta\lambda}{4\pi n}. \tag{19}$$

That is, for a given position x, simply put in the value of the measured phase shift in radians and compute y.

Most likely, δ won't be useful as $\frac{\partial B_x}{\partial x}$ *and* $\frac{\partial B_y}{\partial x}$ because of greater experimental difficulty.

6. EXPERIMENTAL

One of several experiments which verify the theory was conducted by Stolar Research Corp. over the Otay Mesa tunnel near San Diego. The Stolar device uses two in-line ferrite loop antennas at 50 kHz whose outputs are subtracted to produce a Δ B; thus, a horizontal component of the gradient is measured. Therefore, the response for a traverse over the tunnel should resemble Figure 2, and that's just what occurred. Not only did the method locate the structure, but it also allowed a depth determination at various points along the tunnel by measuring the (changing) location of the peaks in the x (transverse) direction as predicted by theory.

7. SUMMARY

The experiments performed using the gradient method have shown what appears to be a definite advantage over a measurement of the magnetic field itself. This is due to the effect of external radio source concellation; that is, the signal to noise ratio is vastly improved. Not only does the gradient method locate the source, but it also determines the depth. The horizontal gradient component of either the vertical or horizontal magnetic field may be used. It's suggested that the ultimate instrument measure both.

ACKNOWLEDGMENT

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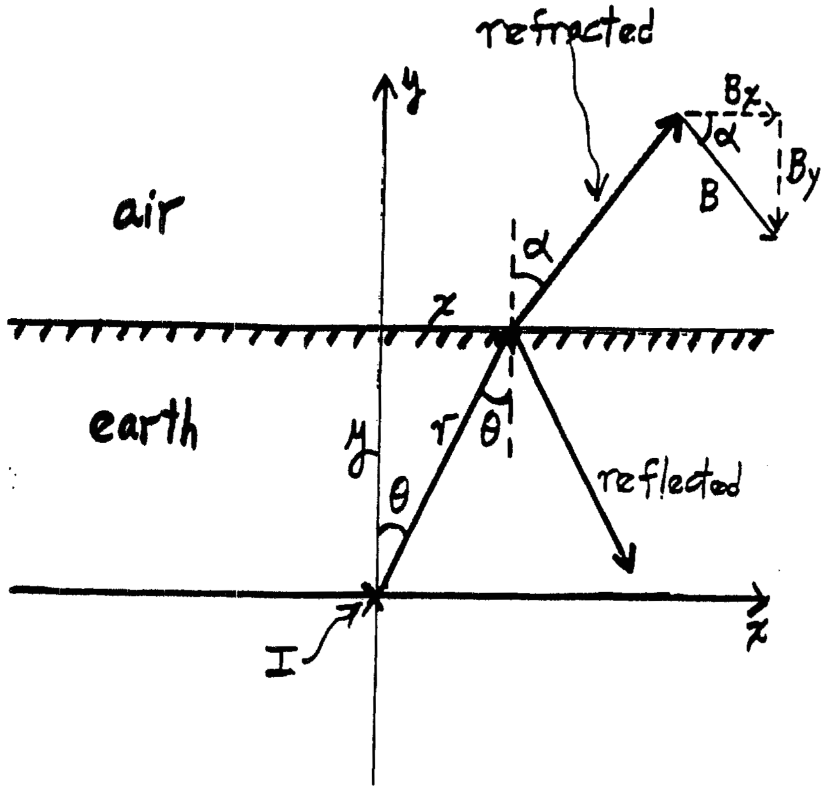

Figure 1. Geometry for a line source of current I buried in the earth of index n at a depth y.

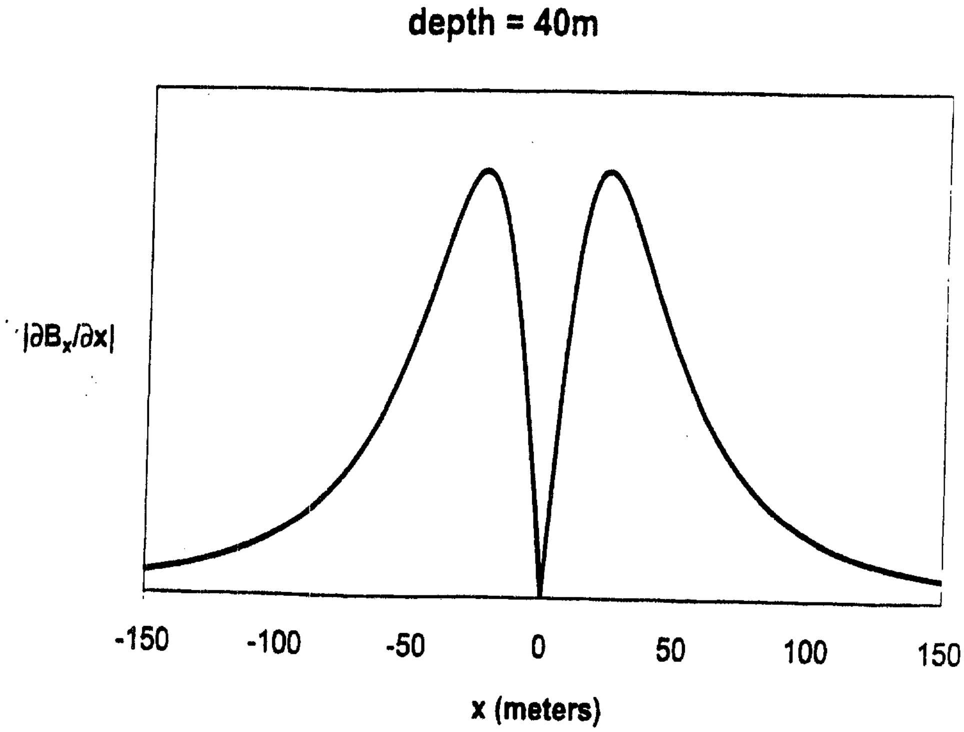

Figure 2. Horizontal variation of B_x for a line source at a depth of 40m in soil with n close to unity.

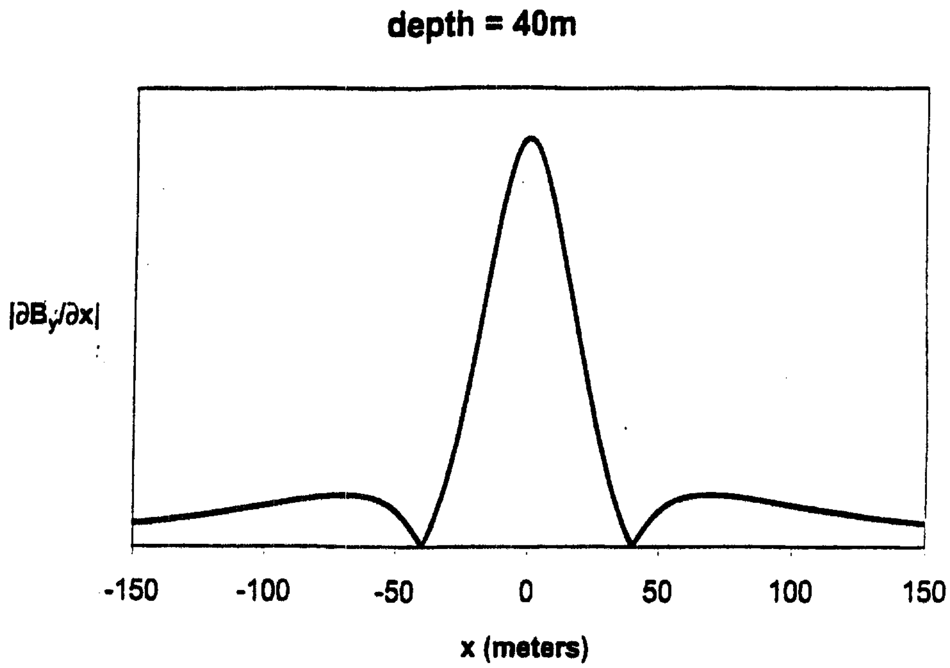

Figure 3. Horizontal variation of B_y for a line source at a depth of 40m in soil with n close to unity.